Differential Equations

Language (minimum required)

Def. Deriving an equation which contains one or more variables for one or more independent variables, is said to be a differential equation (ED). A partial differential equation involving one or more dependent variables of two or more independent variables is called a partial differential equation (PDE).

These equations are classified by type, order and linearity.

- **By type:** whether an ED contains only derived from one or more dependent variables with respect to a single independent variable, the die which is an ordinary differential equation (ODE)
- **In order:** the order for ED or an ODE is the order of the highest derivative in the equation. for example

\[
\frac{d^2 x}{dt^2} + 3 \left( \frac{dx}{dt} \right) - 2x = e^t
\]

**First order**

- **By linearity:** DE of n-th order \(F(x, y, y', \cdots, y^{(n)}) = 0\) is said to be linear \(y, y', \cdots, y^{(n)}\) if \(F\) is linear. This means that an ODE of nth-order equation is linear when

\[
F(x, y, y', \cdots, y^{(n)}) = 0 \text{ is } a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y + a_0(x)y - g(x) = 0
\]

**Initial general Exercises**

1. In the following examples indicate whether it is an ordinary differential equation or is it a partial differential equation.

   1. \[
   \frac{dy}{dx}(x) = y(x), \text{ or } \frac{dy}{dx} = y \cdot \quad y' - y = 0
   \]

   2. \[
   \frac{d^2 y}{dx^2}(x) = e^x + 1, \text{ or } \quad y'' = e^x + 1
   \]

   3. \[
   \frac{d^3 y}{dx^3}(x) + 3 \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} = 1, \text{ or } \quad y''' + 3y'' + 2y' = 1
   \]
4. \( t \frac{d(x)}{dt} + (2 \ln t) x = e^t \), or \( tx' + (2 \ln t) x = e^t \)

5. \( \frac{d^2 z(x)}{dx^2} - z(x) = 0 \), or \( z''(x) - z(x) = 0 \), or \( z' - z = 0 \)

6. \( \left( \frac{dr}{ds} \right)^2 = \sqrt{\frac{d^2 r}{ds^2} + 1} \)

7. \((y'')' + (y')' + 3y = 5x \)

8. \( t^2 x'' + 2tx' + x = \ln(t) \)

9. \( \frac{\partial U(x,y)}{\partial x} + \frac{\partial^2 U(x,y)}{\partial y^2} - U(x,y) = 0 \), or \( U_x + U_y - U = 0 \)

10. \( \frac{\partial^2 U(x,y)}{\partial x^2} + \frac{\partial^2 U(x,y)}{\partial y^2} = 0 \), or \( U_{xx} + U_{yy} = 0 \)

11. \( \frac{\partial^4 Z(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^2 Z(x,y)}{\partial x^2} + \frac{\partial^2 Z(x,y)}{\partial y^2} + Z(x,y) = 0 \)

2. Classify each of the following E. D. as ordinary or partial, and also indicate the order in each case which name (s) is (s) variable (s) independent.

1. \( \frac{dy}{dx} + y = 1 \)

2. \( x^2 dy + y^2 dx = 0 \)

3. \( \frac{d^2 x}{dt^2} = -\frac{1}{x^2} \)

4. \( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0 \)

5. \( \frac{\partial^4 U}{\partial t^2 \partial s^2} + \frac{\partial^2 U}{\partial t \partial s} + U = 0 \)

6. \( \frac{d^4 y}{dt^4} + \left( \frac{d^2 y}{dt^2} \right)^4 + y = 0 \)

7. \( \frac{dr}{ds} = \sqrt{1 + \left( \frac{d^2 r}{ds^2} \right)^2} \)

8. \( \frac{\partial^2 U}{\partial y^2 \partial x} + \frac{\partial^2 U}{\partial x^2} = 0 \)

9. \( \frac{d^2 x}{dt^2} + \left( \frac{d^2 x}{dt^2} \right)^2 \left( \frac{dx}{dt} \right) + x = t + 1 \)

10. \( \frac{d^4 y}{dx^4} + 3 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + y = 5e^{-2x} - 3x^2 + 2 \)

11. \( xy' + xy = 1 - x \)

12. \( \frac{d^2 x}{dy^2} + e^y \frac{dx}{dy} - x = 4y^2 \)
13. \[
\frac{d^2x}{dt^2} + \left(\ln t \right) x = 1
\]

14. \[
t^3 \frac{d^3y}{dt^3} + 2t^2 \frac{d^2y}{dt^2} + t \frac{dy}{dt} + y = \ln t
\]

15. \[
\frac{\partial^2 U}{\partial x \partial y} - \frac{\partial U}{\partial x} - \frac{\partial U}{\partial y} + U = 0
\]

3. Try to find some functions that are solutions to the equations of items 1 and 2 of this guide.

References:

Berkley/Blanchard, "Calculos", Saunders College Publishing
Elsgoltz: Ecuaciones diferenciales y Cálculo variacional. Mir, 3a ed.