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Derivatives of inverse trigonometric functions

You use implicit differentiation to find the derivatives of the inverse trigonometric functions. Because if $f(x)$ it is derivable, its inverse is also differentiable noticed $f^{-1}(x)$, except where its tangents are vertical. This is possible because the graph of a vertex is not derivable funtion or loops, thereby, if the reflected with respect to $y = x$, the graph of its inverse function also has vertices or loops.

The derivative of inverse trigonometric function is noted as: $\frac{d}{dx}\left(sin^{-1}\ x\right)=\frac{d}{dx}\,arc\,sin\,x$, either notation is accepted. Are defined:

$$\frac{d}{dx}\left(sin^{-1}\ x\right)=\frac{d}{dx}\,arc\,sin\,x=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(cos^{-1}\ x\right)=\frac{d}{dx}\,arc\,cos\,x=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(tan^{-1}\ x\right)=\frac{d}{dx}\,arc\,tan\,x=\frac{1}{1+x^2} \qquad \frac{d}{dx}\left(cot\,an^{-1}\ x\right)=\frac{d}{dx}\,arc\,cotan\,x=\frac{-1}{1+x^2}$$

$$\frac{d}{dx}\left(sec^{-1}\ x\right)=\frac{d}{dx}\,arc\,sec\,x=\frac{1}{x\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(csc^{-1}\ x\right)=\frac{d}{dx}\,arc\,csc\,x=\frac{-1}{x\sqrt{x^2-1}}$$

Example 1: Derive $y=\frac{1}{sin^{-1}x}$

Solution: $y=\frac{1}{sin^{-1}x}=\left(sin^{-1}x\right)^{-1}$ hence: $\frac{dy}{dx}=\frac{d}{dx}\left(sin^{-1}x\right)^{-1}=-\left(sin^{-1}x\right)^{-2}\frac{d}{dx}\left(sin^{-1}x\right)\Rightarrow-\frac{1}{\left(sin^{-1}x\right)^2\sqrt{1-x^2}}$

Example 2: Derive $f(x)=x\,arctan\sqrt{x}$ the shape of *f (x)* is a product, therefore apply the appropriate formula, see:

Solution: $f'(x)=1.arctan\sqrt{x}+x\frac{1}{1+\left(\sqrt{x}\right)^2}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)=\frac{\sqrt{x}}{2(1+x)}+arctan\sqrt{x}$

Exercises:

Find $\frac{dy}{dx}$ by implicit, simplify to the maximum extent possible:

1. $y=arctan\sqrt{x}$
2. $y=\sqrt{tan^{-1}x}$
3. $f(x)=sin^{-1}(2x+1)$

4. $g(x)=\sqrt{x^2-1}\,sec^{-1}x$ 5. $h(t)=arc\,cotan\sqrt{t}+cot^{-1}\left(\frac{1}{t}\right)$

6. $y=arctan\sqrt{\frac{1-x}{1+x}}$ 7. $y=cos^{-1}\left(e^{2x}\right)$ 8. $y^2+x^2=1$

9. $2\sqrt{x}+\sqrt{y}=3$ 10. $x^2+x\sqrt{y}-y^2=3xy$ 11. $2\sqrt{x+y}=1+\sqrt{y}$

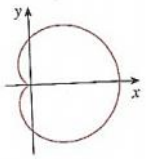
12. $sin\,x+cos\,2\sqrt{y}=3\,sin\,x\,cos\,x^{-1}$ 13. $e^{\sqrt{y}}\,cos\,x+1-sin^{-1}\left(e^{2\sqrt{x}}\right)=0$

For the years, from 14 to 19, use implicit differentiation to find an equation of the tangent to the curve at the given point:

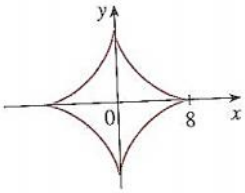
14. $x^2+xy+y^2=3,\quad p=(1,1)$ *ellipse.*

15. $x^2+2xy-y^2+x=2,\quad p=(1,2)$ *hyperbola.*

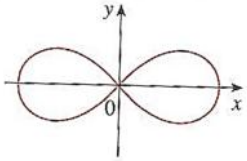
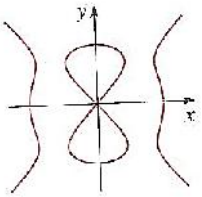
16. $x^2+y^2=\left(2x^2+2y^2-x\right)^2,\quad p=(0,\frac{1}{2})$ *one cardioid.*



17. $x^{2/3}+y^{2/3}=4,\quad p=\left(-3\sqrt{3},1\right)$ *one astroide.*



18. $2(x^2+y^2)^2=25\left(x^2-y^2\right),\quad p=(3,1)$ *one lemnixcata.*



19. $y^2(y^2-4)=x^2\left(x^2-5\right),\quad p=(0,-2)$ *Devil curve.*