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PhD advances Mathematics.

Differential Equations

Language (minimum required)

Def. Deriving an equation which contains one or more variables for one or more independent variables, is said to be a differential equation (ED). A partial differential equation involving one or more dependent variables of two or more independent variables is called a partial differential equation (PDE).

These equations are classified by type, order and linearity.

- *By type: whether an ED contains only derived from one or more dependent variables with respect to a single independent variable, the die which is an ordinary differential equation (ODE)*
- *In order: the order for ED or an ODE is the order of the highest derivative in the equation. for example*

Second order

$$\frac{d^2x}{dt^2}+3\left(\frac{dx}{dt}\right)-2x=e^t$$

First order

- *By linearity: DE of n-th order* $F\left(x,y,y',\cdots,y^{(n)}\right)=0$ *is said to be linear* $y,y',\cdots,y^{(n)}$ *if F is linear. This means that an ODE of nth-order equation is linear when* $F\left(x,y,y',\cdots,y^{(n)}\right)=0$ *is* $a_n(x)y^{(n)}+a_{n-1}(x)y^{(n-1)}+\cdots+a_1(x)y'+a_0(x)y-g(x)=0$

Initial general Exercises

1. In the following examples indicate whether it is an ordinary differential equation or is it a partial differential equation.

1. $\frac{dy(x)}{dx}=y(x),\ or,\ \frac{dy}{dx}=y \quad ó, \quad y'-y=0$

2. $\frac{d^2y(x)}{dx^2}=e^x+1,\ or, \quad y''=e^x+1$

3. $\frac{d^3y(x)}{dx^3}+3\frac{d^2y}{dx^2}+2\frac{dy}{dx}=1,\ or,\ y'''+3y''+2y'=1$

4. $t\dfrac{d(x)}{dt}+(2\ln t)x=e^{t}$, *or*, $tx'+(2\ln t)x=e^{t}$

5. $\dfrac{d^4z(x)}{dx^4}-z(x)=0$, *or*, $z^4(x)-z(x)=0$, *or*, $z^4-z=0$

6. $\left(\dfrac{dr}{ds}\right)^3=\sqrt{\dfrac{d^2r}{ds^2}+1}$

7. $(y'')^3+(y')^4+3y=5x$

8. $t^2x''+2tx'+x=\ln(t)$

9. $\dfrac{\partial U(x,y)}{\partial x}+\dfrac{\partial^2U(x,y)}{\partial y}-U(x,y)=0$, *or*, $U_x+U_y-U=0$

10. $\dfrac{\partial^2 U(x,y)}{\partial x^2}+\dfrac{\partial^2U(x,y)}{\partial y^2}=0$, *or*, $U_{xx}+U_{yy}=0$

11. $\dfrac{\partial^4 Z(x,y)}{\partial x^2\partial y^2}+\dfrac{\partial^2Z(x,y)}{\partial x^2}+\dfrac{\partial^2Z(x,y)}{\partial y^2}+Z(x,y)=0$

2. Classify each of the following E. D. as ordinary or partial, and also indicate the order in each case which name (s) is (s) variable (s) independent.

1. $\dfrac{dy}{dx}+y=1$

2. $x^2dy+y^2dx=0$

3. $\dfrac{d^2x}{dt^2}=-\dfrac{1}{x^2}$

4. $\dfrac{\partial^2U}{\partial x^2}+\dfrac{\partial^2U}{\partial y^2}=0$

5. $\dfrac{\partial^4U}{\partial t^2\partial s^2}+\dfrac{\partial^2U}{\partial t\partial s}+U=0$

6. $\dfrac{d^3y}{dt^3}+\left(\dfrac{d^2y}{dt^2}\right)^4+y=0$

7. $\dfrac{dr}{ds}=\sqrt{1+\dfrac{d^2r}{ds^2}}$

8. $\dfrac{\partial^3U}{\partial y^2\partial x}+\dfrac{\partial^2U}{\partial x^2}=0$

9. $\dfrac{d^2x}{dt^2}+\left(\dfrac{d^3x}{dt^3}\right)\left(\dfrac{dx}{dt}\right)+x=t+1$

10. $\dfrac{d^3y}{dx^3}+3\dfrac{d^2y}{dx^2}+3\dfrac{dy}{dx}+y=5e^{-2x}-3x^2+2$

11. $yy'+xy=1-x$

12. $\dfrac{d^2x}{dy^2}+e^{y}\dfrac{dx}{dy}-x=4y^2$

13. $\frac{d^2x}{dt^2}+\left(ln\left|t\right|\right)x=1$

14. $t^3\frac{d^3y}{dt^3}+2t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+y=ln\,t$

15. $\frac{\partial^2 U}{\partial x\partial y}-\frac{\partial U}{\partial x}-\frac{\partial U}{\partial y}+U=0$

3. Try to find some functions that are solutions to the equations of items 1 and 2 of this guide

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