Prof. Enrique Mateus Nieves
PhD en Educación Matemática

Indefinite Integrals

An indefinite integral $\int f(x)\,dx$ is a family of functions of the form $\int f(x)\,dx = F(x) + c$, where $F'(x) = f(x)$ and c It represents an arbitrary constant. Hence: if $\{ F(x) + c \}$ It is the set of all functions whose differentials are $f(x)\,dx$, It is also the set of all functions whose derivative is $f(x)$. Therefore, the anti-drift is considered as the operation to determine the set of all functions with a derivative. While a remarked definite integral of the form: $\int_a^b f(x)\,dx$ is a number representing an area in square units. The relationship between them is provided by part two of the Fundamental Theorem of Calculus (FTC): if f It is a continuous function on $[a,b]$, then $\int_a^b f(x)\,dx = \int f(x)dx \Big|_a^b$

Note:

i. We call "exact value of the indefinite integral" to expressions of the form of the following example:

$$\int_0^2 \left(2x^3 - 6x + \frac{3}{x^2+1} \right) dx = \frac{1}{2}x^4 - 3x^2 + 3\tan^{-1}x \Big|_0^2 = -4 + 3\tan^{-1}2 \text{ und}^2$$

The expression $-4 + 3\tan^{-1}2$ represents the "exact value of the integral" If we want a decimal approximation, just use the calculator for a value close to $\approx -0{,}67855$

ii. iiWe must adopt a convention that when a formula for a general indefinite integral this is valid only within a range provided. For instance:

$\int \frac{1}{x^2}\,dx = -\frac{1}{x} + c$ This is only valid in the intervals $(-\infty, 0)$ o en $(0, +\infty)$ meaning that the general anti-derivative of the function $f(x) = \frac{1}{x^2},\ x \neq 0$ is:

$$F(x) = \begin{cases} -\frac{1}{x} + c_1 & \text{if } x < 0 \\ -\frac{1}{x} + c_2 & \text{if } x > 0 \end{cases}$$

Example:

Find the general indefinite integral for $\int \left(10\ x^4 - 2\ sec^2 x\right) dx$

Solution:

$$\int \left(10\ x^4 - 2\ sec^2 x\right) dx = 10 \int x^4\ dx - 2 \int sec^2 x\ dx$$

$$= 2x^5 - 2\ tan\ x + c$$

Theorem total change

The integral of a gear ratio is the total change $\int_a^b F'(x)\, dx = F(b) - F(a)$

Application example: a particle moves along a straight line so that its velocity at time t is $v(t) = t^2 - t - 6$ (mediated in meters per second). Find the displacement of the particle during the time interval elapsed between $1 \le t \le 4;$ also find the distance in this period.

Solution: The equation of the displacement is given by the expression: $\int v(t) = s(t_2) - s(t_1)$, hence the movement is; $s(4) - s(1) = \int_1^4 v(t)\, dt = \int_1^4 \left(t^2 - t - 6\right) dt = -\frac{9}{2}$ this means that the particle moves leftward 4.5 m

Exercise-workshop

Find the following general indefinite integral:

1. $\int x^2 + x^2 dx =$
2. $\int \left(x^4 - \frac{1}{2}x^3 + x - \frac{2}{3} \right) d\ x =$
3. $\int \sqrt{x^3} + \sqrt[3]{x^2}\ dx$
4. $\int \frac{x^3 - 2\sqrt{x}}{x^2}\ dx$
5. $\int h\left(h^2 + 2\right) dh$
6. $\int \left(1 + tan^2 x\right) dx$
7. $\int x^2 + 1 + \frac{1}{x^2 + 1}\ dx$
8. $\int \left(b - Csc\ \ b\ cotan\ \ b\ \right) db$